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The effect of cooling on supersonic boundary-layer stability

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For a number of applications it is important to know the location of the boundary-layer transition from laminar to turbulent. At present it is generally recognized that the onset of turbulence is directly connected with the loss of stability of the initial laminar flow. In the overwhelming majority of cases experimental data on the influence of various factors upon the transition location agree well with the calculated data concerning the influence of these factors on the boundary-layer stability, i.e. the theory of stability may be used successfully to predict various experimental dependencies.

The boundary-layer stability and the transition are considerably affected by heat transfer from the surface of the streamlined body. But, in this case, experimental data on the transition do not always correspond to the results of the stability theory. In particular, experimental works concerning the effect of cooling of the model surface on the supersonic boundary layer transition yield contradictory results (see e.g. Gaponov & Maslov 1980; Morkovin 1969). Some of the contradictions were removed by Demetriades (1978) and Lysenko & Maslov (1981), but on the whole the problem cannot be considered solved, primarily owing to the fact that many theoretical results have not yet been experimentally confirmed.

In the present paper the experimental study of development of small natural disturbances in the boundary layer of a cooled flat plate for Mach numbers $M = 2$, 3 and 4 is described. It confirms the main conclusions of the linear theory of hydrodynamic stability concerning the fact that surface cooling: (i) stabilizes the first-mode disturbances; (ii) destabilizes the second-mode disturbances; (iii) may lead to the region of unstable frequencies of the first mode being divided into two; (iv) does not affect the interaction of acoustic waves and the supersonic boundary layer.

1. Introduction

Three main modes of oscillation may occur in a supersonic boundary layer:

(1) first-mode disturbances;

(2) second- and higher-mode disturbances;

(3) forced acoustic disturbances.

These oscillation modes are considered, with special attention paid to the influence of cooling of the streamlined surface upon them.

First-mode disturbances. This oscillation mode was discovered in theoretical work by Lees and Lin (see Lin 1955). Disturbances of this mode in a boundary layer may become unstable, their instability being caused by viscosity. These oscillations are analogous to the well-known Tollmien–Schlichting waves of an incompressible fluid. Phase velocities $C > 1-1/(M\cos\chi)$ ($M \equiv$ Mach number, $\chi \equiv$ angle of propagation of disturbance) and moderate frequencies $F \sim 10^{-4}$ ($F = \alpha C/Re \equiv$ non-dimensional frequency parameter, $\alpha \equiv$ wavenumber of disturbance, $Re \equiv$ Reynolds number) are characteristic of them.

The shape of velocity, density and temperature profiles of the averaged flow affect the propagation of first-mode disturbances considerably. With cooling of the model surface these profiles change in such a way that the flow becomes more stable with respect to this wave mode, and there are ranges of disturbance propagation angles for which it is possible to achieve complete stabilization of the flow with the aid of cooling (i.e. the boundary layer will contain no other than damped disturbances).

These main conclusions are in the first works of Lees and Lin, who made use of the asymptotic analysis of stability equations. Subsequently they were confirmed and specified in works of Gaponov & Maslov (1980), and Mack (1969, 1975*a*) on the basis of a numerical solution of the stability equations. In addition it was discovered that surface cooling may result in the stability region on the (α, Re) diagram being separated into two subregions (i.e there were two neutral curves) (Gaponov & Maslov 1980; Maslov 1972), while in the case of three-dimensional disturbances there may occur closed instability regions that, with decreasing surface temperature, quickly contract to a point and disappear completely (Maslov 1974).

Second-mode disturbances. Oscillations of this type are a variety of acoustic resonance in a shear flow. Their characteristic phase velocities are somewhat greater than those of the first mode, and the values of their frequency parameter are considerably greater. For a boundary layer on a heat-insulated surface they may occur at $M > 2.2$. Instability of this type was discovered in the works of Mack (1964) and Gill (1965), and was investigated in considerable detail by Mack (1969). It was established that tilted disturbances of this type ($\chi \neq 0$) are more stable in comparison with two-dimensional disturbances ($\chi = 0$) and that model-surface cooling exerts a weak destabilizing influence on them.

Forced acoustic disturbances. Disturbances of this type correspond to sound propagation in a moving medium. Phase velocities of propagation of such waves are $C < 1 - 1/(M \cos\chi)$. Lees and Lin (see Lin 1955) mentioned the existence of these disturbances in their earlier works. They were investigated more thoroughly by Mack (1975*a*) and Gaponov (1977). According to these works, external acoustic disturbances are considerably amplified in a supersonic boundary layer. The amplification factors of acoustic disturbances increase with increasing Mach number. Studies on the effect of cooling upon waves of this type showed that it suppresses somewhat the disturbances in the boundary layer, but on the whole this influence is very weak.

Experimental work. Comparison with the theory. The number of experiments concerning research of the development of small perturbations in supersonic boundary layers is not numerous. Practically all of them have been carried out on heat-insulated (adiabatic) surface models. Laufer & Vrebalovich (1960) were the first to carry out experimental work that confirmed the existence of first-mode disturbances.

Kendall (1966), by introducing artificially excited disturbances in the course of testing a heat-insulated model at $M = 4.5$, obtained the amplification rates both for three-dimensional first-mode disturbances and two-dimensional second-mode disturbances, and they were in very good agreement with the data of Mack (1969). In the later work of Kendall (1975) for $M = 8.5$ at constant value of $T_w = 0.6$ this agreement was not so good (the amplification rates differ by a factor of 2). Here T_w is the temperature factor, the ratio of wall temperature to adiabatic wall temperature.

Interaction of sound with the supersonic boundary layer was considered experimentally by Kendall (1975), Lebiga, Maslov & Pridanov (1977), and Gaponov *et al.*

(1977). All major conclusions concerning the behaviour of disturbances in a boundary layer on a heat-insulated model were borne out. In the work of Gaponov *et al.* good agreement between the calculation and the experiment on the study of the disturbance propagation in a supersonic boundary layer was received, these disturbances being caused by an external acoustic field.

The work of Demetriades (1978), conducted on a cone at $M = 8$, with the temperature factor $T_w = 0.93$ and 0.48, was the first (and up to now the only one) to show experimentally that for second-mode disturbances a decrease in T_w results in an increase in the amplification rates and shift of the stability diagram towards larger frequencies. However, no comparison with the calculated data is made in this work.

Thus at the present time a certain amount of progress has been achieved in the theoretical study of wave processes in supersonic boundary layers on cooled surfaces, while there are practically no experiments (except measurements by Demetriades (1978)) checking the theoretical conclusions. The present work undertakes to conduct such experiments and to compare their results with the calculations.

2. Equipment and experimental technique

The experiments were performed in the wind tunnel T-325 of the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the USSR Academy of Sciences. The test section was 200 mm high, 200 mm wide and 600 mm long. The measurements were carried out in the boundary layer of a stainless-steel flat plate 350 mm long, 200 mm wide and 9 mm thick. The leading edge of the plate was bevelled at an angle of 20° and its bluntness was $b \approx 0.02$ mm (for $M = 3.0$, $b \approx 0.05$ mm). The plate was fixed rigidly to the walls of the tunnel and was placed at zero angle of attack.

In the experiments the model-surface temperature factor varied within the range $0.3 < T_w < 1.0$. The model was cooled with premixed liquid and gaseous nitrogen. Ten coolant normal-to-flow passages with a cross-section 10 mm × 6 mm were made in the plate, each of these passages having an opening in the bottom of the model. The cooling fluid was fed through the passages and exhausted through the openings into the test section of the wind tunnel. The cross-sectional opening of each passage was changed in such a way as to achieve as uniform a temperature of the whole cooled section as possible. An example of the temperature distribution along the plate surface of this model is given in Lysenko & Maslov (1981). The length of the cooled section beginning at a distance of 15 mm from the leading edge was 125 mm. The section corresponding to the first 15 mm was cooled at the expense of the thermal conductivity of the metal.

The model-surface temperature was measured with 10 stainless-steel–constantan thermocouples. The plate served as the common leg, while constantan legs, soldered flush to the plate surface, composed the other legs. The thermal e.m.f. of the thermocouples was registered with a DACQ-2BB multichannel measurement system.

Stability characteristics were measured with a constant-current hot-wire anemometer and a probe with gilded tungsten wire 6 μm in diameter and 1.5 mm long. In the experiments use was made of a DISA root-mean-square voltmeter and Rohde & Schwarz FAT-1 and C4-12 spectrum analysers. In the course of the experiments, the signal at the output of the hot-wire anemometer was recorded on a tape recorder, and then an amplitude–frequency analysis of the obtained recordings was made. The values of the temperature factor were estimated for that section of the surface on which stability characteristics were measured. Neutral stability curves were determined according to the technique described by Lebiga *et al.* (1977).

The results obtained in the work are represented by dimensionless variables. The value $(Re_1/x)^{-\frac{1}{2}}$ is taken for the characteristic linear dimension. Re_1 is the unit Reynolds number calculated per metre. The Reynolds number was then determined from the ratio $Re = (Re_1 x)^{\frac{1}{2}}$; the dimensionless frequency parameter is $F = 2\pi f/Re_1 U$, where f is the disturbance frequency and U is the velocity in a non-disturbed flow. The disturbance amplification rates were determined from the relation: $\alpha_i = -0.5\, \mathrm{d}(\ln A_f)/\mathrm{d}Re$, where A_f is the disturbance amplitude of the frequency f. The measurements (analogous to those by Kendall 1975) were made in a layer where the dimensionless disturbance amplitude (the ratio of the r.m.s. value of the fluctuations of the voltage on the wire of the probe to the value in a non-disturbed flow) reached the maximum.

The distribution of the disturbance amplitude A_f for each frequency along the streamwise coordinate was approximated by the second-order curve by the least-squares method using to the amplitude values obtained in the experiment at nine points, each being spaced at an equal distance from the other in the streamwise direction. The r.m.s. error of this approximation did not exceed 5 %. The disturbance amplification rates were determined for the approximated curve. The error in determining α_i, depending on the operating conditions, varied within the $\pm 0.1 \times 10^{-3}$ (for $Re = 1360$ at $M = 3.0$) to $\pm 0.4 \times 10^{-3}$ (for $Re = 780$ at $M = 4.0$ and for $Re = 700$ at $M = 2.0$) range.

The work of Lysenko & Maslov (1981) showed that, at frost formation on the model, the dependence of the transition Reynolds numbers on the temperature factor was no longer monotonous (a so-called 'reverse' of the transition takes place), the investigations in the present work were conducted at the vapour concentration of the air in the T-325 test section $C = (0.16–0.20) \times 10^{-3}$, which corresponded to a relative air humidity before drainage of $C_h = 0.15–0.31$. As a result, there was no frost on the model during the experiment.

The experiments were conducted at Mach numbers 2, 3 and 4 and for a $(5–50) \times 10^6\ \mathrm{m}^{-1}$ range of unit Reynolds numbers.

3. On the comparison of the experimental results with the calculations

In the present work, as well as in the majority of similar works (excluding the measurements by Demetriades 1960; Kendall 1966; and particularly Laufer & Vrebalovich 1960), study was made of the propagation of natural disturbances arising owing to some cause or other in a boundary layer. Generally, these disturbances represent a set of time and space components. Fourier-transforming in time or the use of narrow-band filters makes it possible to determine the signal-power spectrum of the hot-wire anemometer corresponding to a certain frequency band. Neither separation of the spatial components nor isolation of the disturbances propagating under certain angles is made in the experiment, and it is not quite clear how to perform them in the case of natural disturbances. Thus some integral-over-direction signal corresponding to a certain frequency is measured in the experiment.

The other limitation characteristic of experiments of this kind is that neither the wavelengths nor the phase velocities of the disturbances are measured. The lack of these data hampers the identification of the disturbances. In this case a comparison with the calculated results (even if approximate) may by helpful. Therefore in this work some calculations have been made in order to single out, without laying any claim to a quantitative comparison, the frequency regions and the main behavioural tendencies of this or that type of disturbances with model-surface cooling.

The theory studies the development of individual components of the disturbance spectrum corresponding not only to a certain frequency but also propagating under a certain angle. What share these components comprise in the signal of the hot-wire anemometer in a particular measurement is not clear. As a rule (see Gaponov & Maslov 1980; Kendall 1975) in such cases the experimental data are compared with the calculations for the most rapidly growing (according to the theory) disturbances, in the belief that it is they that ultimately determine the summary signal. This comparison cannot be regarded as quite correct.

Taking this into account, in the present work comparison was made between the experimental data and the calculated data for different disturbance propagation angles.

The disturbance-amplification rates were determined in terms of the numerical integration of the stability equations in the Dunn–Lin approximation (see Lin 1955). Mack (1969) found good agreement of the calculated amplification rates for three-dimensional disturbances for the Dunn–Lin system with a more complete system of the eighth-order stability equations. Besides, studies carried out in recent years (Gaponov 1980; El-Hady & Nayfeh 1979) have shown that the correction to the amplification rates of the inclined disturbances by adding dissipative terms to the Dunn–Lin equations is of the same order as the correction considering the nonparallelism of the flow in the boundary layer. For these reasons, and considering the objective of this work, the Dunn–Lin system was chosen for carrying out the calculations.

The parameters of the mean flow were defined on the basis of the numerical integration of the equations for the boundary layer of the plate.

The stability equations were integrated numerically using the method of orthogonalization. In the calculations use was made of Prandtl number $\sigma = 0.72$, ratio of specific heats $\gamma = 1.41$, and the law of viscosity as a function of the temperature according to Sutherland (see e.g. Hirschfelder, Curtis & Bird 1954).

In solving the problem, the frequency was assumed to be real, while $\beta_i/\alpha_i = \beta_r/\alpha_r$ (Mack 1975*b*), where $\beta = \beta_r + i\beta_i$ is the wavenumber in the transverse direction. Then $\chi = \arctan(\beta_r/\alpha_r)$, where χ is the angle of wave incidence relative to the basic flow.

4. Results

The first series of investigations were carried out at Mach number $M = 2$ and Reynolds number $Re = 700$ ($Re_1 \approx 5 \times 10^6\ \mathrm{m}^{-1}$). The measurement results for the case of a heat-insulated surface are set out in figure 1. The values of the disturbance amplification rates $-\alpha_i$ and the dimensionless frequency F are plotted along the axes. The results of the calculation for several first-mode disturbance propagation angles are plotted with dashed lines. The experimental data of Laufer & Vrebalovich (1960) for $M = 2.2$ and $Re = 793$ are given in the same figure. From the comparison it is evident that the disturbances observed in the experiment rank, according to their frequency range, among first-mode disturbances. The best agreement between the experimental data and the theoretical results is observed for $\chi = 42°$.

Mention should be made that calculations based on a more complete system of stability equations and taking account of the non-parallelism of the flow in the boundary layer may be expected to yield somewhat different results (though still close to the obtained ones). Another important consideration is that the experimental data

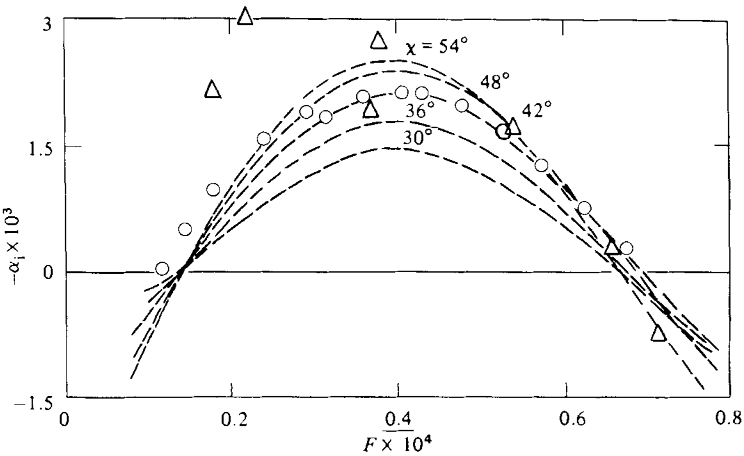

FIGURE 1. Comparison of the experimentally obtained disturbance amplification rates on a heat-insulated model for $M = 2.0$ and $Re = 700$ with numerical results at different angles of disturbance propagation: ○, experiment; ---, calculated, △ is the experiment of Laufer & Vrebalovich (1960) for $M = 2.2$ and $Re = 793$.

are in good agreement with the calculations for some one angle of incidence of the wave front relative to the flow. Probably, this has to do with the fact that the discrepancy of the calculated data in this case, at the variation of χ in the range 30°–50°, is not large. It is also probable that this is a manifestation of a 'receptivity' of the boundary layer to external disturbances.

According to Mack (1975*a*) and Kendall (1975), acoustic waves are the main source of first-mode disturbances for $M \geqslant 2$. As shown by the measurements of V. A. Lebiga, in the T-325 wind tunnel acoustic waves radiated by a turbulent boundary layer of the walls of the nozzle and test section fall on the model, at $M = 2$, precisely at the angle $\chi = 42°$.

Figure 2 illustrates the effect of cooling on the first-mode disturbances. In this case calculations are given for $\chi = 42°$ only. For other disturbance-propagation angles the influence of cooling is analogous to that given in figure 2. Decreasing surface temperature results in a monotonic decrease of the amplification rates and a reduction of the range of unstable frequencies. It is also seen that the linear theory of hydrodynamic stability predicts correctly that the disturbance amplification rates change with decreasing temperature.

Figure 3 presents data for $M = 2$ and $Re = 1600$ ($Re_1 = 25 \times 10^6\ \mathrm{m}^{-1}$). The dashed lines are the calculated results for $\chi = 42°$. Both in the theory and the experiment the frequency range of unstable disturbances ($-\alpha_i > 0$) became considerably narrower, in comparison with figure 2, and was shifted towards the beginning of coordinates.

Figure 3 also presents the disturbance-amplification rates for low frequencies ($F \sim 0.01 \times 10^{-4}$). This frequency band, for the considered values of M and Re, corresponds to the most intensive interaction of acoustic waves with the boundary layer (Gaponov *et al.* 1977), the third type of fluctuations out of those mentioned in §1. As follows from the figure, decreasing temperature does not influence (within the error of estimation) disturbances of this type, which agrees with the conclusions of Mack (1975*a*).

For high frequency values ($F > 0.15 \times 10^{-4}$) experimental data begin to show considerable differences from the calculation for first-mode disturbances, and, with

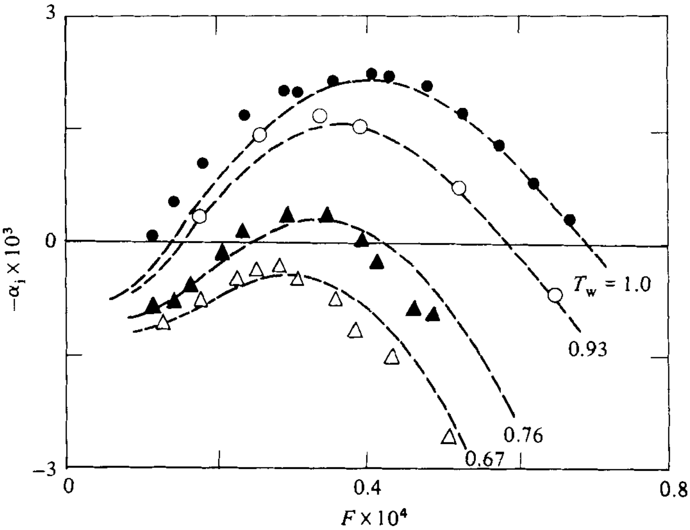

FIGURE 2. Experimental and numerical amplification rates of first-mode disturbances as functions of the frequency parameter for $M = 2.0$ and $Re = 700$: ---, calculated, $\chi = 42°$; ●, ○, ▲, △, experiment.

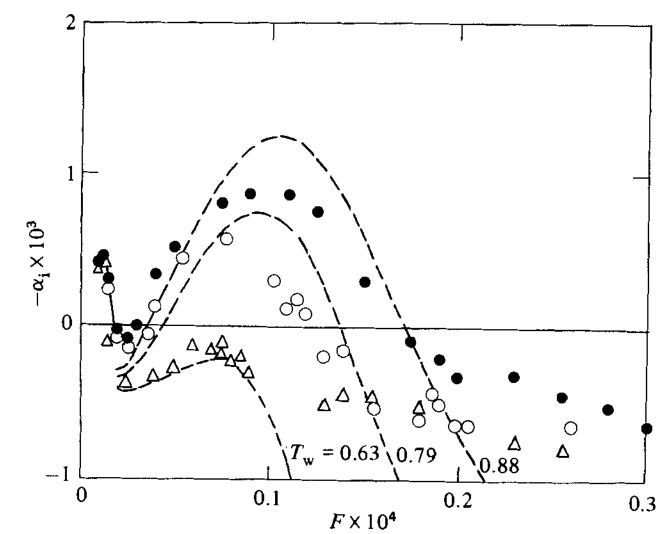

FIGURE 3. Experimental and numerical disturbance amplification rates as functions of the frequency parameter for $M = 2.0$ and $Re = 1600$: ---, calculated, $\chi = 42°$.

increasing F, the experimental data received for different values of T_w become closer. Probably this is owing to the fact that, for high frequencies, first-mode disturbances are intensively damped and become considerably smaller than fluctuations due to the interactions of acoustic waves with the boundary layer. And it is because of this that the amplification rates obtained in the experiment with cooling at high frequencies are determined by the interaction of acoustic fluctuations with the boundary layer, on which cooling has a weak influence.

Figure 4 shows the distribution of the disturbance amplitude A_f, normalized to the first maximum value, with the dimensionless frequency $F = 0.052 \times 10^{-4}$ along the streamwise coordinate X for $Re_1 = 26 \times 10^6\ \mathrm{m}^{-1}$ and different values of T_w. The first

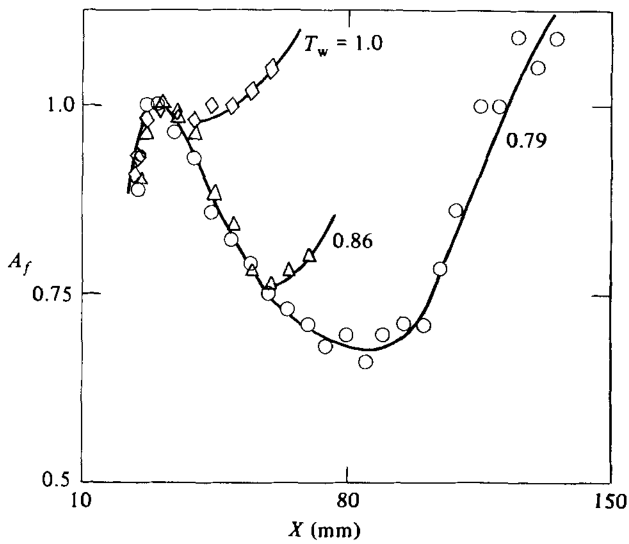

FIGURE 4. Distribution of the amplitude of disturbance along the streamwise coordinate for $M = 2.0$, $Re_1 = 26 \times 10^6\ \mathrm{m}^{-1}$ and $F = 0.052 \times 10^{-4}$.

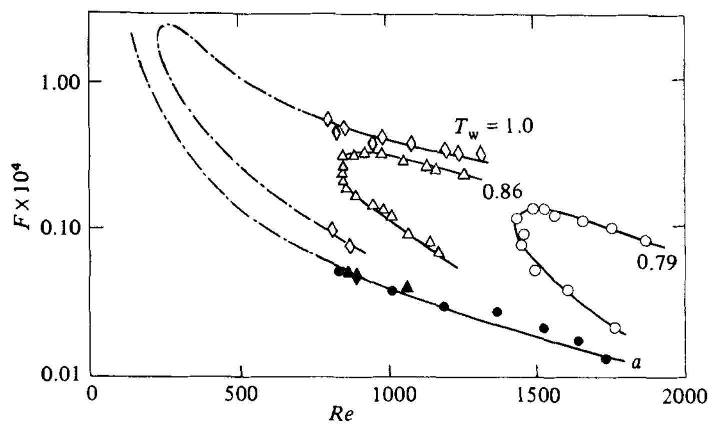

FIGURE 5. Experimental curves of neutral stability for first-mode disturbances with different cooling of the model and the line of maximum amplification of acoustic waves (a) for $M = 2.0$.

maximum of this dependence is determined by the interaction of sound and the boundary layer, while the minimum corresponds to the onset of the amplification of the total signal of the first-mode disturbances and sound (Lebiga *et al.* 1977; Gaponov *et al.* 1977). Its position depends on the distribution and relation between the amplitudes of forced and first-mode oscillations.

It is seen from the figure that, while the position of the minimum of the signal (determining the lower branch of the neutral stability curve) is shifted with surface cooling in the direction of larger X, the position of the acoustic maximum remains (within the error of estimation) unchanged, which confirms the conclusions made earlier.

Figure 5 presents the neutral stability curves, obtained experimentally for $M = 2$ at three values of the temperature factor ($T_w = 1.00$, 0.86 and 0.79) and $Re_1 = 26 \times 10^6\ \mathrm{m}^{-1}$. Here the value T_w corresponds to the average temperature of the

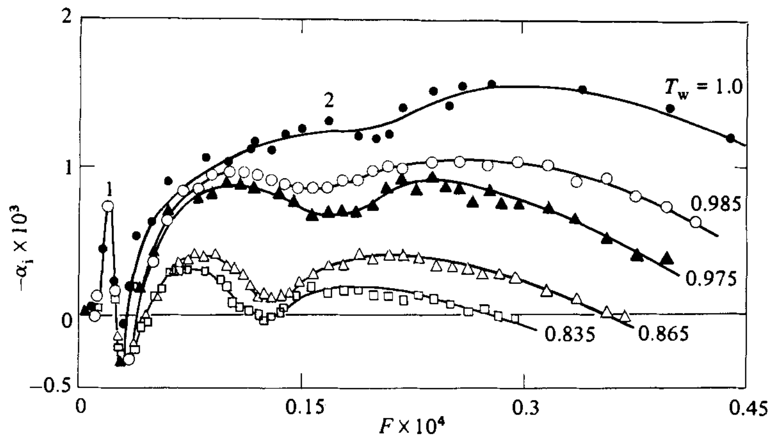

FIGURE 6. Experimental amplification rates as a function of the frequency parameter for $M = 3.0$ and $Re = 1360$: (1) forced acoustic disturbances; (2) first mode.

whole section under cooling, starting with the leading edge. F is the dimensionless frequency and Re is the Reynolds number. These curves separate the regions of disturbance amplification from those of disturbance damping observed in the experiment. For $Re < 800$ at $T_w = 1.0$ the curve of neutral stability (dash–dotted line) is plotted on the basis of averaging numerous experimental data obtained by Lebiga, Maslov & Pridanov (1977, 1979) in T-325 on a flat heat-insulated plate at different values of unit Reynolds number. It is seen from the figure that with model-surface cooling the neutral stability curves for the first-mode disturbances are shifted monotonically towards the region of larger Reynolds numbers, and the band of instability frequencies is steadily decreasing. The position of the line of maximum amplification of acoustic waves (curve a) does not change with cooling.

The next series of experiments were carried out at Mach number $M = 3$ and Reynolds number $Re = 1360$ ($Re_1 = 50 \times 10^6$ m^{-1}). The results are presented in figure 6. It is seen that at consistent cooling two 'humps' occur on the curve $-\alpha_i = \Phi(F)$, which become prominent at large cooling. This pattern corresponds to the separation of the instability region (first mode) into two, for $M = 3$, at cooling. As follows from the figure, the second region (at larger F) at cooling is stabilized quicker than the first.

This behaviour of first-mode disturbances at surface cooling for $M = 3$ was discovered by means of calculations in Maslov (1972, 1974). However, in these series we did not manage to make a comparison of the data obtained experimentally with the calculated results in a proper way. In order to increase the Reynolds number, the experiments were carried out at a large value of $Re_1 = 50 \times 10^6$ m^{-1}, and as a result (because of the necessity to conduct measurements in the linear stage of disturbance propagation) at low values of streamwise coordinate X (X = 20–40 mm), that is, on the section with a considerable temperature gradient (see Lysenko & Maslov 1981, figure 1). Besides, in the present series of experiments the ratio $Re_b/Re = 0.04$ (Re_b is the Reynolds number calculated from the thickness of the leading edge) was several times larger than in other series. As a result, the averaged flow was practically unknown. Another complexity in comparing the theoretical findings with the experiment arose owing to the fact that theoretical works predicted a division of the first-mode instability region into two for small angles of disturbance propagation,

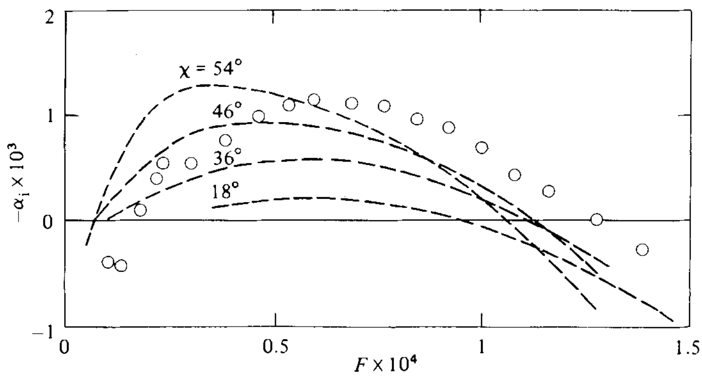

FIGURE 7. Comparison of experimentally obtained amplification rates of first-mode disturbances for $M = 4.0$, $Re = 780$ and $T_w = 0.83$ with the numerical results at different angles of disturbance propagation: ○, experiment; ---, calculated.

$\chi < 40°$. In this case the Dunn–Lin approximation yields only qualitatively correct results (quantitative results differ substantially when the Dunn–Lin system is complemented with dissipative terms; see Brown 1967). In the case of the flat plate in a flow without any pressure or temperature gradients, the instability region is separated in two, to the Dunn–Lin approximation, at Reynolds numbers larger than 1360.

The next series of experiments was carried out at $M = 4$ and $Re = 780$ ($Re_1 = 6 \times 10^6\ \mathrm{m}^{-1}$). In accordance with Mack (1969), at the Mach numbers $M > 3.6$ second-mode (high-frequency) disturbances occur, while at $M \geqslant 4$ second-mode disturbances become more unstable as compared with first-mode disturbances. In the present series of experiments we managed to watch the effect of cooling both on the first- and second-mode disturbances (figures 8 and 10).

Figure 7 presents, by way of example, for $T_w = 0.83$ the comparison of the first-mode disturbance-amplification rates obtained in the experiment with the calculated results for different angles of wave inclination χ. The calculated results are the closest to the experimental data for $\chi = 46°$. Similarly, as for $M = 2$, cooling (figure 8) has a stabilizing influence on first-mode disturbances (the disturbance amplification rates are steadily decreasing, so is the unstable frequency band) and does not affect the sound–boundary-layer interaction. The results of the calculation for $\chi = 46°$ are plotted in figure 8 with dashed lines. A dash–dotted line is used for $T_w = 0.46$ and $\chi = 60°$. Generally, the experimental data are observed to agree qualitatively with the theoretical results.

The best agreement between experiment and theory was observed at $M = 2$ and low Reynolds numbers. With increasing M and Re the coincidence was worse, and at $M = 4$ and $Re = 1700$ it was not possible to find any calculated dependence (at some definite angle) close enough to the experimental one.

Theoretical values for the amplification rates close enough to the experimental ones may be obtained (as the integral characteristics) only in the presence in the boundary layer of waves with different angles of inclination simultaneously. That is, if some specified (initial) wave direction is characteristic of small values of the Reynolds number, then, with increasing Re, disturbances with other angles of χ are rapidly amplified.

Figure 9 shows the effect of cooling on the first-mode disturbance-amplification

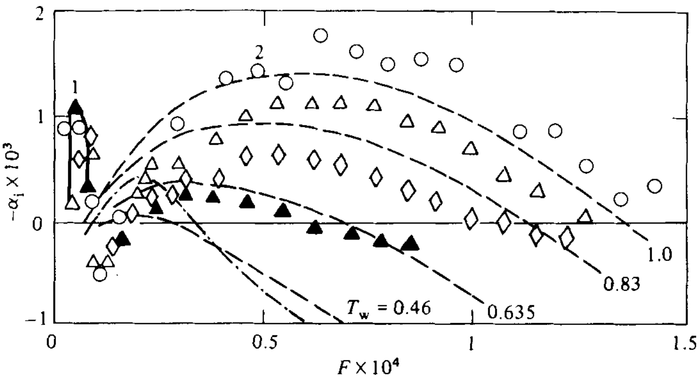

FIGURE 8. Experimental and numerical amplification rates of disturbances as functions of the frequency parameter for $M = 4$ and $Re = 780$: (1) forced acoustic disturbances (experiment); (2) first mode; ○, △, ◇, ▲, experiment; ---, calculated, $\chi = 46°$; —·—·, calculated, $\chi = 60°$.

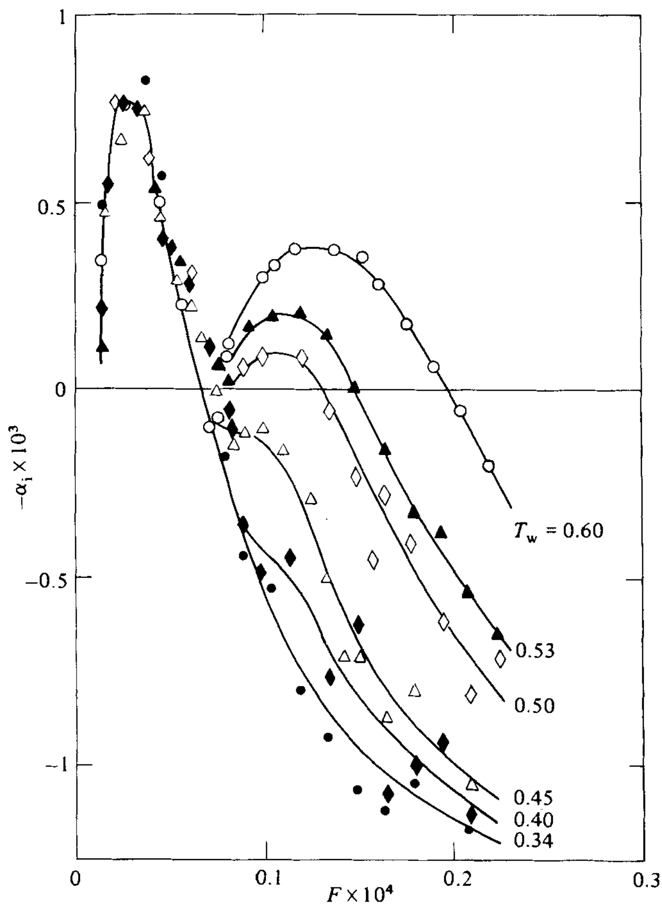

FIGURE 9. Experimental amplification rates of disturbances as functions of the frequency parameter for $M = 4$ and $Re = 1700$.

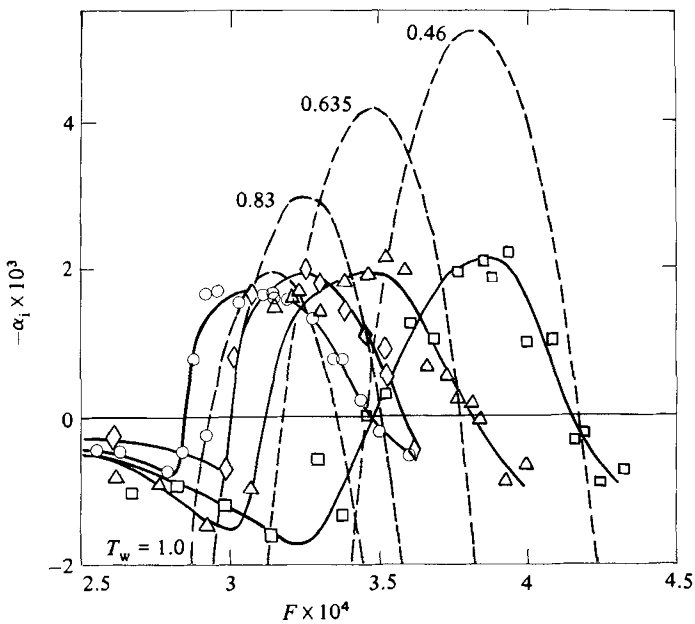

FIGURE 10. Comparison of experimental and numerical amplification rates of second-mode disturbances for $M = 4$ and $Re = 780$: ---, calculated, $\chi = 0$; ——, experiment (○, $T_w = 1.0$; ◇, 0.83; △, 0.635; □, 0.46).

rates as well as on the interaction of acoustic waves and boundary layer at $M = 4$ and $Re = 1700$. These data are interesting because of the following.

According to Mack (1969) and Gaponov (1977), with increasing Mach number the role of acoustic disturbances grows, and the intensity of fluctuations generated by the external acoustic field in the boundary layer is, at $M = 4$, higher, and the contribution of the 'amplification rates' due to the interaction of acoustic waves with boundary layer to the total (experimentally registered) values is considerably larger than at $M = 2$. The dependence $-\alpha_i = \Phi(F)$ obtained experimentally at deep cooling ($T_w = 0.34$) seems to coincide fully with the curve specified by the interaction of acoustic waves and boundary layer. In the rest the results presented in figure 9 are fully analogous to the data obtained for $M = 2$, presented in figure 3.

Cooling has a destabilizing influence on second-mode disturbances (figure 10). The disturbance-amplification rates are increased, and the unstable frequency band is also somewhat increased; in this case it is shifted into the region of larger frequencies. The calculated results for the most unstable – two-dimensional disturbances ($\chi = 0$) – are plotted with dashed lines.

It is worth noting that if for heat-insulated surface ($T_w = 1.0$) the maximum amplification rates are approximately the same for both first- and second-mode disturbances, at any cooling the maximum amplification rates for the second mode are higher than for first.

As is seen from figure 10, for second-mode disturbances the calculated and experimental bands of unstable frequencies coincide well with each other, and, qualitatively, the theory predicts correctly the behaviour of the disturbance amplification rates with decreasing surface temperature.

5. Conclusions

In the present work it has been found, both experimentally and by way of calculations, that *surface cooling*:

(i) stabilizes first-mode disturbances – the range of unstable frequencies decreases, the amplification rates decrease, and the neutral stability curves are shifted to the domain of larger Reynolds numbers;

(ii) destabilizes second-mode (high-frequency) disturbances – the unstable frequency region expands and is shifted to the region of larger frequencies, while the amplification rates grow;

(iii) may lead to the region of unstable frequencies of the first mode being divided into two;

(iv) does not affect (within the error of estimation) the interaction of acoustic waves and supersonic boundary layer (forced acoustic disturbances).

The experimental results are in a qualitative agreement with the predictions of the theory of hydrodynamic stability.

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